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THE PRYTZ PLANIMETER.

By DR. A. R. CRATHORNE, University of Illinois.

The object of this article is to call attention to a little known but very interesting and simple mathematical instrument, the Prytz planimeter, or to give its more common name — the hatchet planimeter. It is easily made from a piece of stiff wire which is bent into the form shown in Fig. 1. One end of the wire is ground to a point and the other end is flattened into a chisel edge. The sharp point and this edge should be in the same plane. For a given area a planimeter which is longer than the longest diameter should be used.

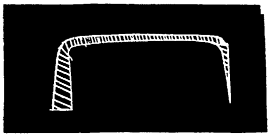

Fig. 1.

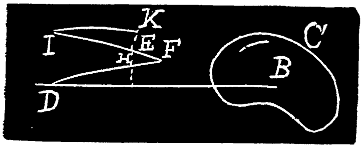

Fig. 2.

To obtain the area of a closed curve with this planimeter a straight line of indefinite length is drawn from the approximate center of gravity in any direction (Fig. 2). The pointed end of the instrument is placed on the center of gravity and the chisel edge on the straight line (at *A* in the figure). The legs of the planimeter must at all times be perpendicular to the plane of the area. From its position *B*, the tracing point is now moved along the straight line to the intersection with the boundary curve and then around the area in the direction indicated and back to *B*. The edged end of the instrument, upon which a slight pressure is brought to bear, traces out the curve *ADEFHIK* and when the tracing point has returned to *B*, will take the position *K*. The product of the length *AK* into the length of the planimeter will be the approximate area of the given curve (see equation [7]).

This instrument was invented some fifteen years ago by Captain Prytz of the Danish Army, who published an account of his invention in the English magazine, *Engineering*, Vol. 72, page 813. A detailed analytical discussion of its theory was given by M. F. W. Hill in the *Philosophical Magazine* for 1894. In the *Bulletin de l' Acadêmie Imperiale des Sciences de St. Petersbourg*, 1903, Professor Kriloff of the Russian Naval Academy discussed the instrument from the geometrical standpoint and gave a very elementary and simple explanation of its theory.

This theory depends on the well known theorem:* The total area *Z* swept out by a straight line *AB* moving in a plane is given by the formula

$$[1] \qquad Z = ls + (\tfrac{1}{2}l^2 - al)(\theta_2 - \theta_1),$$

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*See Chapter XIV, Gibson's *Calculus*.

where l is the length of the moving line; s, the total normal displacement of a point P of the line; a, the distance AP; and θ_1, θ_2 are the initial and final values of θ, the angle made by the moving line with some fixed line. If, in particular, the point A moves around a closed curve C', while at the same time B makes a complete circuit of the curve C which lies entirely outside of C', then $\theta_2=\theta_1$, and we have

$$Z=C-C'=ls. \tag{2}$$

The usual conventions as to the signs of the areas in question hold. Areas covered twice in opposite directions by the line are zero, and an area described counter clock wise is positive. If a small wheel with its axis in the moving line be attached at P, the arc through which it turns will give us the normal displacement s. We will call this wheel the measuring wheel. It should be noticed that the position of the point P does not enter into equation [2]. In measuring areas with most planimeters the area of the curve C' is a constant of the instrument. In the case where the point corresponding to A moves backwards and forwards on a curve this constant is zero. In the well known Amsler polar planimeter the curve C' is a circle, and in some other instruments it is a straight line. The area C is the area to be measured.

In the Prytz planimenter there is no fixed curve C', but instead we have a curve which depends upon the curve whose area we are measuring. Referring to figure 2, we see that as the point B moves in its path, the edge A moves in its curve of pursuit from A to K. Now turn the planimeter horizontally about the point B until it is in the initial position AB. The curve corresponding to the curve C' is now a closed curve $ADEFHIKA$. The total area swept out by the line AB is equal to the algebraic sum of the area C and the areas ADE, EFH, and HIK. The other parts of the plane swept by the line are covered twice in opposite directions and hence do not enter into the algebraic sum. Putting in the proper signs we have

$$\text{Total area swept out}=C-ADE+EFH-HIK. \tag{3}$$

From equation [2] the total area swept out is measured by the product of the length l of AB into the length of arc s through which a measuring wheel at A on the line AB would have turned. During the motion of the tracing point this wheel does not turn at all, for the direction of motion is perpendicular to the edge of the wheel. But in turning the instrument about B from the position KB into AB, this wheel will turn through an arc which is equal in length to $l\phi$ where ϕ is the angle KBA. Or we have

$$s=l\phi. \tag{4}$$

The total area swept out by AB is ls or $l^2\phi$. This gives the exact equation,

$$[5] \qquad l^2\phi = C - ADE + EFH - HIK.$$

If the starting point for tracing the figure be taken as above (*i. e.* at the center of gravity of the area), the algebraic sum of the three areas enclosed by the curve of pursuit and the arc KA will be very nearly zero and we have the approximate equation

$$[6] \qquad c = l^2\phi = l.\text{arc}\,KA.$$

If the angle ϕ be small, say less than 20°, the arc KA can be replaced by its chord, and we have

$$[7] \qquad c = l.KA.$$

If an area whose longest diameter is four inches or less, be measured with a ten inch planimeter, the error is very small and is about equal to the error made in finding the area of an equivalent rectangle by measuring the sides with a scale. The error due to the non-alignment of the edge and tracing point can be eliminated by tracing the curve in opposite directions and finding the mean of the two results.

An improved planimeter of this type has a small chisel-edged wheel instead of the chisel edge.

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EXISTENCE OF A MINIMUM OF A QUADRATIC FUNCTION.

By T. H. HILDEBRANDT, The University of Chicago.

Suppose we have a quadratic function in n variables,

$$F[x_1, \ldots, x_n] = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}\, x_i\, x_j + \sum_{i=1}^{n} b_i\, x_i + c,$$

of which we know that

$$F[x_1, \ldots, x_n] \geqq 0,$$

for all values of the variables $x_1, \ldots, x_n$. For instance, such a function is

$$\int_0^{2\pi} [\phi(x) - \sum_{i=1}^{n} (a_i \cos ix + b_i \sin ix)]^2 dx,$$